Violation of the Holographic Shear Viscosity Bound in Strongly Coupled Isotropic Plasmas with Einstein Gravity Dual

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We revisit the AdS/CFT calculations of the shear viscosity of isotropic, strongly coupled plasmas with Einstein gravity dual by emphasizing the fact, which was overlooked in the previous literatures, that the shear viscosity is a fourth-rank tensor. We show that since the shear viscosity tensor has to be regular at the horizon, its indices, in the bulk, should be raised and lowered by using the bulk metric components evaluated at the horizon. We find that indeed the holographic (Kovtun-Son-Starinets) shear viscosity bound is saturated by the 'two indices up and two indices down' shear viscosity tensor to entropy density ratio $\frac{\eta^{j}{}_{i}{}^{j}{}_{i}}{s}$ but it is violated by both the 'four indices down' $\frac{\eta_{jiji}}{s}$ and the 'four indices up' $\frac{\eta^{jiji}}{s}$. Specifically, for the isotropic, strongly coupled $\mathcal{N}=4$ super-Yang-Mills plasma with Einstein gravity dual, we find that $\frac{\eta_{jiji}}{s}=\frac{1}{4}\pi^3T^4R^4$, and $\frac{\eta^{jiji}}{s}=\frac{1}{4\pi^5T^4R^4}$.

Introduction. At the center of the application of AdS/CFT techniques [1–3] for calculating the transport coefficients of strongly coupled plasmas [4, 5][6] is the holographic (Kovtun-Son-Starinets) shear viscosity bound which was conjectured by Kovtrun, Son, and Starinets [7] after observing the fact that the shear viscosity to entropy density ratio takes the value $\frac{1}{4\pi}$ for large class of strongly coupled isotropic quark gluon plasmas with Einstein (second-derivative) gravity duals [8]. No violation of the bound has been found so far in its regime of validity, i.e., isotropic and Einstein gravity, even though, violation of the bound has been found in anisotropic systems [9–11] [12] and higher-derivative gravities [13][14]. In addition, non-universal value has been found in other anisotropic systems [15] [16]. In this paper, we report the finding that the bound can actually be violated within its 'regime of validity' in holographic models which exhaust all the previously studied ones, of course, including the isotropic, strongly coupled $\mathcal{N}=4$ super-Yang-Mills plasma with Einstein gravity dual.

The key observation for our finding comes from realizing the fact that the boundary energy-momentum tensor operators $T^{\mu\nu}$, $T_{\mu\nu}$, and $T^{\mu}{}_{\nu}$ have different sources in the bulk, i.e., the boundary values of $h_{\mu\nu}$, $h^{\mu\nu}$, and $h_{\mu}{}^{\nu}$, respectively. (Note that each source satisfies different equations of motions in the bulk.) Hence, the two-point functions $\langle T^{\mu\nu}T^{\mu\nu}\rangle$, $\langle T_{\mu\nu}T_{\mu\nu}\rangle$, and $\langle T^{\mu}{}_{\nu}T^{\mu}{}_{\nu}\rangle$ are different. And, consequently the shear viscosity tensors that we'll extract from them, using Kubo's formula, are different. Which means that we'll get three different shear viscosity tensors

$$\eta^{ijij}=\lim_{\omega\to 0}\frac{1}{2\omega}\int dtd\mathbf{x}e^{i\omega t}\langle[T^{ij}(x),T^{ij}(0)]\rangle, \quad (1)$$

$$\eta_{ijij}=\lim_{\omega\to 0}\frac{1}{2\omega}\int dtd\mathbf{x}e^{i\omega t}\langle[T_{ij}(x),T_{ij}(0)]\rangle, \quad (2)$$

$$\eta^{i}{}_{j}{}^{i}{}_{j}=\lim_{\omega\to 0}\frac{1}{2\omega}\int dtd\mathbf{x}e^{i\omega t}\langle[T^{i}{}_{j}(x),T^{i}{}_{j}(0)]\rangle, \quad (3)$$

where the indices i, j stand for the spatial coordinates x, y, z, and $i\neq j$, and the indices μ, ν stand for t, x, y, z.

However, since, in the bulk, the sources $h_{\mu\nu}$, $h^{\mu\nu}$, and $h_{\mu}{}^{\nu}$ have simple relationships amongst each other, i.e., $h_{\mu\nu}=g_{\mu\mu}g_{\nu\nu}h^{\mu\nu}=g_{\nu\nu}h_{\mu}{}^{\nu}$, the two point functions should also have this kind of simple relationships, i.e., $\langle T^{\mu\nu}T^{\mu\nu}\rangle=g^{\mu\mu}g^{\nu\nu}g^{\mu\mu}g^{\nu\nu}\langle T_{\mu\nu}T_{\mu\nu}\rangle=g^{\nu\nu}g^{\nu\nu}\langle T^{\mu}{}_{\nu}T^{\mu}{}_{\nu}\rangle$, as it can be derived by recalling that

$$\begin{aligned}\langle T^{\mu\nu}T^{\mu\nu}\rangle&=\frac{\delta^2 S_{on-shell}}{\delta h_{0\mu\nu}(q)\delta h_{0\mu\nu}(q')},\\ \langle T_{\mu\nu}T_{\mu\nu}\rangle&=\frac{\delta^2 S_{on-shell}}{\delta h_0^{\mu\nu}(q)\delta h_0^{\mu\nu}(q')},\\ \langle T^{\mu}{}_{\nu}T^{\mu}{}_{\nu}\rangle&=\frac{\delta^2 S_{on-shell}}{\delta h_{0\mu}{}^{\nu}(q)\delta h_{0\mu}{}^{\nu}(q')}\end{aligned} \quad (4)$$

and, noting that

$$\begin{aligned}\langle T^{\mu\nu}T^{\mu\nu}\rangle&=\frac{\delta^2 S_{on-shell}}{\delta h_{0\mu\nu}(q)\delta h_{0\mu\nu}(q')},\\ &=\frac{\delta^2 S_{on-shell}}{\delta g_{\mu\mu}g_{\nu\nu}h_0^{\mu\nu}(q)\delta g_{\mu\mu}g_{\nu\nu}h_0^{\mu\nu}(q')},\\ &=g^{\mu\mu}g^{\nu\nu}g^{\mu\mu}g^{\nu\nu}\frac{\delta^2 S_{on-shell}}{\delta h_0^{\mu\nu}(q)\delta h_0^{\mu\nu}(q')},\\ &=g^{\mu\mu}g^{\nu\nu}g^{\mu\mu}g^{\nu\nu}\langle T_{\mu\nu}T_{\mu\nu}\rangle,\end{aligned} \quad (5)$$

and, also

$$\begin{aligned}\langle T^{\mu\nu}T^{\mu\nu}\rangle&=\frac{\delta^2 S_{on-shell}}{\delta h_{0\mu\nu}(q)\delta h_{0\mu\nu}(q')},\\ &=\frac{\delta^2 S_{on-shell}}{\delta g_{\nu\nu}h_{0\mu}{}^{\nu}(q)\delta g_{\nu\nu}h_{0\mu}{}^{\nu}(q')},\\ &=g^{\nu\nu}g^{\nu\nu}\frac{\delta^2 S_{on-shell}}{\delta h_{0\mu}{}^{\nu}(q)\delta h_{0\mu}{}^{\nu}(q')},\\ &=g^{\nu\nu}g^{\nu\nu}\langle T^{\mu}{}_{\nu}T^{\mu}{}_{\nu}\rangle,\end{aligned} \quad (6)$$

where $h_{0\mu\nu}(q)=h_{\mu\nu}(q,u=0)$.

Therefore, the simple relationships amongst the shear viscosity tensors, in the bulk, will be $\eta^{ijij}=$

$g^{ii}g^{jj}g^{ii}g^{jj}\eta_{ijij} = g^{jj}g^{jj}\eta^{i}{}_{j}{}^{i}{}_{j}$ which can be interpreted as raising and lowering of their indices using the bulk metric components g_{ii} and g_{jj}. And, as we'll see explicitly soon, due to the regularity condition at the horizon $u = 1$, the bulk metric components which we'll use to lower and raise the indices should be evaluated at the horizon $u = 1$, i.e., $g_{ii} = g_{ii}(u = 1)$. It should also be noted that once we calculated the shear viscosity tensors in the bulk either directly by using their corresponding two point functions or using the simple relationships amongst them, we can start to raise and lower the indices of each tensors by using the Minkowski metric at the boundary.

For example, all the previously carried out AdS/CFT calculations of the ratio of $\eta^{j}{}_{i}{}^{j}{}_{i}$ to the entropy density s of the strongly coupled isotropic plasmas with Einstein gravity dual have resulted, see for example [8],

$$\frac{\eta^{j}{}_{i}{}^{j}{}_{i}}{s} = \frac{1}{4\pi}g_{ii}g^{jj} = \frac{1}{4\pi}, \tag{7}$$

hence, a motivation for the holographic (Kovtun-Son-Starinets) conjecture [7].

So, raising the lower indices of $\eta^{j}{}_{i}{}^{j}{}_{i}$ in (7) using the bulk metric components $g_{ii}(u = 1)$ evaluated at the horizon $u = 1$, and rearranging, we'll have,

$$\frac{\eta^{jiji}}{s} = \frac{1}{4\pi}g^{jj}(1)g^{ii}(1) = \frac{1}{4\pi^5T^4R^4}. \tag{8}$$

where $R^4 = g^2_{YM}N_c\ell_s^4$.

Similarly,

$$\frac{\eta_{jiji}}{s} = \frac{1}{4\pi}g_{ii}(1)g_{jj}(1) = \frac{1}{4}\pi^3T^4R^4. \tag{9}$$

Therefore, we observe that the shear viscosity tensor to entropy density ratios $\frac{\eta^{jiji}}{s}$ and $\frac{\eta_{jiji}}{s}$ are functions of the temperature T unlike $\frac{\eta^{j}{}_{i}{}^{j}{}_{i}}{s}$ which is a constant.

The Shear Viscosity Tensor. The shear viscosity is a fourth-rank tensor η^{ijij} which relates the gradients in the local fluid velocity u^i to the dissipative part of the stress tensor T^{ij}_D as [15]

$$T^{ij}_D = -\eta^{ijij}u_{ij}, \tag{10}$$

where u_{ij} is the symmetric combination of u_i, i.e., $u_{ij} = u_{ji} = \frac{1}{2}(\partial_j u_i + \partial_i u_j)$, and the indices $i \neq j \neq k$ run over the spatial coordinates x, y and z.

Similarly, we can define the dissipative part of the stress tensor T^D_{ij} as [19]

$$T^D_{ij} = \eta_{ijij}u^{ij}. \tag{11}$$

In most AdS/CFT literatures, see for example [8], the dissipative part of the stress tensor T^D_{ij} is defined as

$$T^D_{ij} = -\eta u_{ij}, \tag{12}$$

which, after restoring the indices for η, can be re-written as

$$T^D_{ij} = -\delta_{jj}\delta^{ii}\eta^{j}{}_{i}{}^{j}{}_{i}u_{ij}. \tag{13}$$

Finally, in the early calculations of the shear viscosity using gravity, for example by Damour [20], the dissipative part of the stress tensor was defined as, see section IV of reference [20],

$$T^{j}_{D\,i} = 2\eta u^{i}{}_{j}, \tag{14}$$

which, after restoring the indices for η, can be re-written as

$$T^{j}_{D\,i} = 2\eta^{j}{}_{i}{}^{j}{}_{i}u^{i}{}_{j}. \tag{15}$$

AdS/CFT, Membrane Paradigm, Holographic RG Flow and the Shear Viscosity Tensor. As shown in [8], later in [17], and more recently in [18] the relevant equations for gravitational shear mode fluctuations can be mapped onto an electromagnetic problem. Consider a metric perturbation of the form

$$g_{iN}(r) \to g_{iN}(r) + g_{ii}h^{i}{}_{N}(x_M \neq i) \tag{16}$$

in the bulk spacetime [6]

$$\begin{aligned} ds^2 &= g_{MN}dx^M dx^N = g_{tt}dt^2 + g_{ii}dx^i dx^i + g_{uu}du^2 \\ &= \frac{\pi^2T^2R^2}{u}\left(-f(u)dt^2 + dx^2 + dy^2 + dz^2\right) \\ &+ \frac{R^2}{4f(u)u^2}du^2 \end{aligned} \tag{17}$$

where $T = r_0/\pi R^2$ is the Hawking temperature, $R^4 = g^2_{YM}N_c\ell_s^4$, and we have introduced $u = r_0^2/r^2$ and $f(u) = 1 - u^2$. The horizon corresponds to $u = 1$, the spatial infinity to $u = 0$. Indices: $\{L, M, N,\}$ run over the full 5-dimensional bulk; $\{i, j, k\}$ run over all spatial coordinates x, y, and z. The Einstein summation convention will apply only for the indices $\{L, M, N,\}$ but not for $\{i, j, k\}$. Comparing this to the standard problem of Kaluza-Klein dimensional reduction along the i spatial direction, setting $A^i_N \equiv h^{i}{}_{N}$, and using the gauge $h_{NN} = h_{uN} = 0$, the Einstein-Hilbert action

$$S_{\mathrm{bulk}} = \frac{1}{2\kappa^2}\int d^5x\sqrt{-g}\,R, \tag{18}$$

after expanding it to second order in the gravitational shear mode fluctuation $h^{i}{}_{N}$, with gravitational coupling $\frac{1}{2\kappa^2} = \frac{1}{16\pi G} = \frac{N_c^2}{8\pi^2R^3}$, can be mapped onto Maxwell's action for the gauge fields A^i_N, with an effective gauge coupling $\frac{1}{g^2_{eff_i}} = \frac{1}{2\kappa^2}g_{ii} = \frac{1}{2\kappa^2}g_{xx} = \frac{1}{2\kappa^2}g_{yy} = \frac{1}{2\kappa^2}g_{zz}$ [8, 17]

$$S_{eff} = -\frac{1}{4}\int d^5x\mathcal{N}^{MMNN}_{ii}F^i_{MN}F^i_{MN}, \tag{19}$$

where

$$F^i_{MN} = \partial_M A^i_N - \partial_N A^i_M, \tag{20}$$
$$A^i_N = h^i{}_N = g^{ii} h_{iN}(t,u,k \neq N), \tag{21}$$
$$\mathcal{N}_{ii}^{MMNN}(u) = \frac{1}{2\kappa^2} g_{ii}\sqrt{-g} g^{MM} g^{NN}. \tag{22}$$

The effective action for A^i_N with the effective gauge coupling g_{eff_i} can be further mapped on to an action for scalar fields $\psi^i{}_j \equiv A^j_i$

$$S_{eff} = -\frac{1}{2}\int d^5x \mathcal{N}_{ii}^{MMjj} \partial_M \psi^i{}_j \partial_M \psi^i{}_j. \tag{23}$$

which upon variation gives the equation of motion for the shear mode gravitational fluctuations $\psi^i{}_j$

$$\partial_M(\mathcal{N}_{ii}^{MMjj}(u)\partial_M \psi^i{}_j) = 0. \tag{24}$$

Integrating by parts the bulk action (23), and using the equation of motion (24), we'll be left with the on-shell boundary action

$$S_{eff} = -S_B[\epsilon], \tag{25}$$

where the boundary action at $u = \epsilon$, $S_B[\epsilon]$, is

$$S_B[\epsilon] = -\frac{1}{2}\int_{u=\epsilon} d^4x \mathcal{N}_{ii}^{uujj} \psi^i{}_j \partial_u \psi^i{}_j. \tag{26}$$

And, the canonical conjugate momentum along the radial direction is

$$\Pi^j{}_i = \frac{\delta S_B}{\delta \psi^i_j} = -\mathcal{N}_{ii}^{uujj} \partial_u \psi^i{}_j. \tag{27}$$

In terms of $\Pi^j{}_i$ (27) the equation of motion (24) can be re-written, in the momentum space, as

$$\partial_u \Pi^j{}_i = -\big(\mathcal{N}_{ii}^{ttjj}\omega^2 + \mathcal{N}_{ii}^{kkjj} q_k^2\big)\psi^i{}_j. \tag{28}$$

The shear viscosity tensor with two indices up and two indices down is defined by $\eta^j{}_i{}^j{}_i \equiv \frac{\Pi^j{}_i}{i\omega\psi^i{}_j}$, and taking its first derivative with respect to ϵ, we'll get

$$\partial_\epsilon \eta^j{}_i{}^j{}_i = \frac{\partial_u \Pi^j{}_i}{i\omega\psi^i{}_j} - \frac{\Pi^j{}_i \partial_u \psi^i{}_j}{i\omega(\psi^i{}_j)^2}. \tag{29}$$

Then, using (28) and (27) in (29), we'll find the holographic RG flow equation for $\eta^j{}_i{}^j{}_i$ to be

$$\partial_\epsilon \eta^j{}_i{}^j{}_i = i\omega\Big(\frac{(\eta^j{}_i{}^j{}_i)^2}{\mathcal{N}_{ii}^{uujj}} + \mathcal{N}_{ii}^{ttjj}\Big) + \frac{i}{\omega}\mathcal{N}_{ii}^{kkjj} q_k^2. \tag{30}$$

It should be noted from (30) that the holographic RG flow of $\eta^j{}_i{}^j{}_i$ is trivial in the hydrodynamics limit $\omega = 0$ and $q_k = 0$, i.e., $\eta^j{}_i{}^j{}_i(\epsilon = 1) = \eta^j{}_i{}^j{}_i(\epsilon = 0) = \eta^j{}_i{}^j{}_i$. And, the initial data at the horizon is provided by requiring regularity at the horizon $\epsilon = 1$ [17]. Since $\frac{1}{\mathcal{N}_{ii}^{uujj}}$ and $\mathcal{N}_{ii}^{ttjj}$ diverge at the horizon $\epsilon = 1$, for the solution of (30) to be regular at the horizon $\epsilon = 1$, the right hand side of (30) has to vanish at $\epsilon = 1$. From which we recover, the frequency and momentum independent result

$$\begin{aligned}\eta^j{}_i{}^j{}_i(\epsilon = 1) &= \sqrt{-\mathcal{N}_{ii}^{uujj}(1)\mathcal{N}_{ii}^{ttjj}(1)} \\ &= \frac{1}{2\kappa^2}\sqrt{\frac{g(1)}{g_{uu}(1)g_{tt}(1)}}\, g_{ii}(1) g^{jj}(1) \\ &= \frac{\pi}{8} N_c^2 T^3.\end{aligned} \tag{31}$$

The same result as (31) can be found by using the membrane paradigm approach [8, 17].

In addition, the same result as (31) can also be found by directly applying AdS/CFT [6]. However, we should note that the indices of the Green's function in Eq. 6.11 of reference [6] are placed imprecisely. Therefore, if we restore the omitted indices on the component of the shear viscosity tensor in Eq. 6.12 of reference [6], and correct the placement of the indices on the Green's function in Eq. 6.11 of reference [6], we'll have

$$\begin{aligned}\eta^y{}_x{}^y{}_x &= \lim_{\omega\to 0}\frac{1}{2\omega}\int dtd\mathbf{x}e^{i\omega t}\langle[T^y{}_x(x), T^y{}_x(0)]\rangle \\ &= -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,(G^y{}_x{}^y{}_x(\omega, q_z = 0)), \\ &= -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,(\frac{\delta^2 S_{on-shell}}{\delta h^x{}_{y0}(q)\delta h^x{}_{y0}(q')}), \\ &= -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,(\frac{\delta^2 S_{on-shell}}{\delta g^{xx}(1)h_{xy0}(q)\delta g^{xx}(1)h_{xy0}(q')}), \\ &= -\lim_{\omega\to 0}\frac{1}{\omega}(-\frac{N_c^2T^2}{16}(2\pi T\omega)) \\ &= \frac{\pi}{8}N_c^2T^3.\end{aligned} \tag{32}$$

where $h^x{}_{y0}(q) = h^x{}_y(q, u = 0)$, and we notice that (32) is exactly equal to (31) as expected. And, our choice to evaluate $g^{xx}(1)$ at the horizon $u = 1$ in (32) is justified, since otherwise $h_{xy0}(q) = g_{xx}(u)h^x{}_{y0}(q)$ diverges at the boundary $u = 0$. This is equivalent to requiring the shear viscosity tensor to be regular at the horizon $u = 1$, a fact which we used to derive (31). And, rearranging (32), we'll have

$$\begin{aligned}g^{xx}(1)g^{xx}(1)\eta^y{}_x{}^y{}_x &= -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,(\frac{\delta^2 S_{on-shell}}{\delta h_{xy0}(q)\delta h_{xy0}(q')}), \\ \eta^{yxyx} &= -\lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\,(\frac{\delta^2 S_{on-shell}}{\delta h_{xy0}(q)\delta h_{xy0}(q')}),\end{aligned} \tag{33}$$

and, (33) can be easily generalized to $x = i$ and $y = j$. Therefore, this proves the fact that the indices on the shear viscosity tensor $\eta^j{}_i{}^j{}_i$ should be raised and lowered by using the metric components $g^{ii}(1)$ evaluated at the horizon $u = 1$.

So, for example, raising the lower indices of $\eta^{j}{}_{i}{}^{j}{}_{i}(\epsilon = 1)$ in (31) using $g_{ii}(1)$ as

$$g_{ii}(1)g_{ii}(1)\eta^{jiji}(1) = \frac{1}{2\kappa^2}\sqrt{\frac{g(1)}{g_{uu}(1)g_{tt}(1)}}g_{ii}(1)g^{jj}(1), \tag{34}$$

the shear viscosity tensor with four indices up $\eta^{jiji}(\epsilon = 1)$ will be

$$\begin{aligned}\eta^{jiji}(1) &= \frac{1}{2\kappa^2}\sqrt{\frac{g(1)}{g_{uu}(1)g_{tt}(1)}}g_{ii}(1)g^{jj}(1)g^{ii}(1)g^{ii}(1)\\ &= \frac{1}{2\kappa^2}\sqrt{\frac{g(1)}{g_{uu}(1)g_{tt}(1)}}g^{jj}(1)g^{ii}(1)\\ &= \frac{N_c^2}{8\pi^3 T R^4}.\end{aligned} \tag{35}$$

Similarly,

$$\begin{aligned}\eta_{jiji}(1) &= \frac{1}{2\kappa^2}\sqrt{\frac{g(1)}{g_{uu}(1)g_{tt}(1)}}g_{jj}(1)g_{ii}(1)\\ &= \frac{1}{8}\pi^5 N_c^2 T^7 R^4\end{aligned} \tag{36}$$

And, using the entropy density s,

$$s = \frac{1}{4G}\sqrt{\frac{g(1)}{g_{uu}(1)g_{tt}(1)}} = \frac{\pi^2 N_c^2 T^3}{2}, \tag{37}$$

the shear viscosity tensor with two indices up and two indices down to entropy density ratio $\frac{\eta^{j}{}_{i}{}^{j}{}_{i}(1)}{s}$ will be

$$\frac{\eta^{j}{}_{i}{}^{j}{}_{i}(1)}{s} = \frac{1}{4\pi}g_{ii}(1)g^{jj}(1) = \frac{1}{4\pi}, \tag{38}$$

while the 'four indices up' shear viscosity tensor to entropy density ratio $\frac{\eta^{jiji}(1)}{s}$ will be

$$\frac{\eta^{jiji}(1)}{s} = \frac{1}{4\pi}g^{jj}(1)g^{ii}(1) = \frac{1}{4\pi^5 T^4 R^4}, \tag{39}$$

where $R^4 = g_{YM}^2 N_c \ell_s^4$.

Similarly, one can show $\frac{\eta_{ijij}}{s}$ to be

$$\frac{\eta_{jiji}(1)}{s} = \frac{1}{4\pi}g_{ii}(1)g_{jj}(1) = \frac{1}{4}\pi^3 T^4 R^4. \tag{40}$$

(39) and (40) are the main results of this paper. Note that at $T = \frac{1}{\pi R}$, $\frac{\eta^{jiji}}{s}(T = \frac{1}{\pi R}) = \frac{\eta_{jiji}}{s}(T = \frac{1}{\pi R}) = \frac{1}{4\pi}$. We should also note that for $g_{ii} = g_{jj} = 1$ which is the case, for example, for the Schwarzschild solution, we find that $\eta^{j}{}_{i}{}^{j}{}_{i} = \eta^{jiji} = \eta_{jiji} = \eta = \frac{1}{2\kappa^2} = \frac{1}{16\pi G}$ which is exactly the same result found by Damour in [20]. Similarly, since $s = \frac{1}{4G}$, $\frac{\eta}{s} = \frac{1}{4\pi}$.

Conclusion. For the first time, we have shown that the holographic (Kovtun-Son-Starinets) shear viscosity bound [7] can be violated in isotropic, strongly coupled $\mathcal{N} = 4$ super-Yang-Mills plasma with Einstein gravity dual. We have found that the shear viscosity tensor (with four indices up or down) to entropy density ratio has strong dependence on the temperature (39)(40).

Even though, we have explicitly derived it only for the shear viscosity tensor (with four indices up or down) to entropy density ratio of the isotropic, strongly coupled $\mathcal{N} = 4$ super-Yang-Mills plasma with Einstein gravity dual, our discussion was general enough to conclude that the shear viscosity tensor (with four indices up or down) to entropy density ratio of all strongly coupled plasmas with gravity dual, for example, the large class of holographic models studied in [8], can be shown to have strong dependence on the temperature.

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